ELEN E3106/4106 Lecture 5

Carriers: Temperature Dependence, Drift, Mobility, and Resistance

Outline

- Temperature effects
- Compensation
- Space charge neutrality
- Mobility & scattering
- Drift velocity & current
- Resistivity

Assignments:

Reading: Streetman and Banerjee §3.3.3-3.3.4, 3.4

Homework 2 due Friday Sept 19th by 5pm

Recap of carrier concentrations

- Recall from Lecture 4, we learned how to get e$^-$ and h$^+$ concentrations at:
  - Any temperature
  - Any doping concentration
  - Any energy level
- We saw that in thermal equilibrium, $np = n_i^2$

* subscript 0 denotes equilibrium

- And, and $n_i^2 = n_0 p_0 = N_c N_v e^{-\frac{E_g}{kT}}$, even when $n_0 \neq p_0$ (doping)
- Given the density of states,

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{\frac{3}{2}} \qquad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{\frac{3}{2}}$$

- And we can conveniently find the carrier concentrations,

$$n_0 = n_i e^{(E_F - E_i)/kT} \qquad p_0 = n_i e^{-(E_i - E_F)/kT}$$

Temperature Dependency of *Intrinsic* Carrier Concentrations

- At any given temperature, *T*:

$$n_i = \sqrt{N_c N_v} e^{-\frac{E_g}{2kT}} \quad cm^{-3}$$

- But it can also be written,

$$n_i(T) = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$$

- What does this tell us?

ni(T) exponentially increases with increasing T

- n_i is very temperature-sensitive! In silicon,
  - While T = 300 → 330 K (10% increase)
  - n_i = ~10^{10} → ~10^{11} cm^{-3} (10x increase)

Sources: E. Pop ECE 340 Slides, Textbook

Visualizing Intrinsic Carrier Concentrations

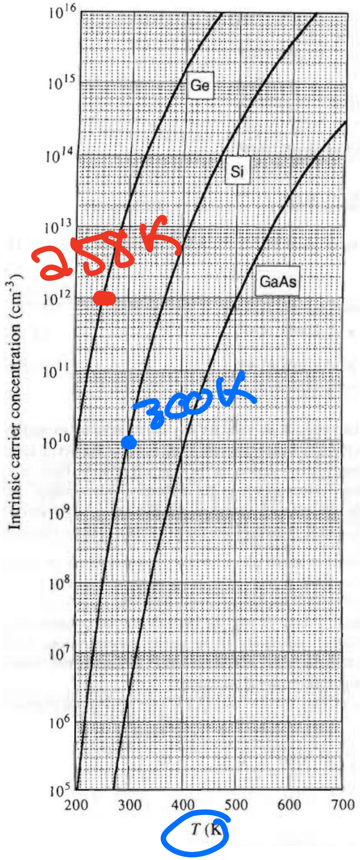
- Plot $\log_{10}$ of n_i vs. T

- What do we expect?

exponential increase

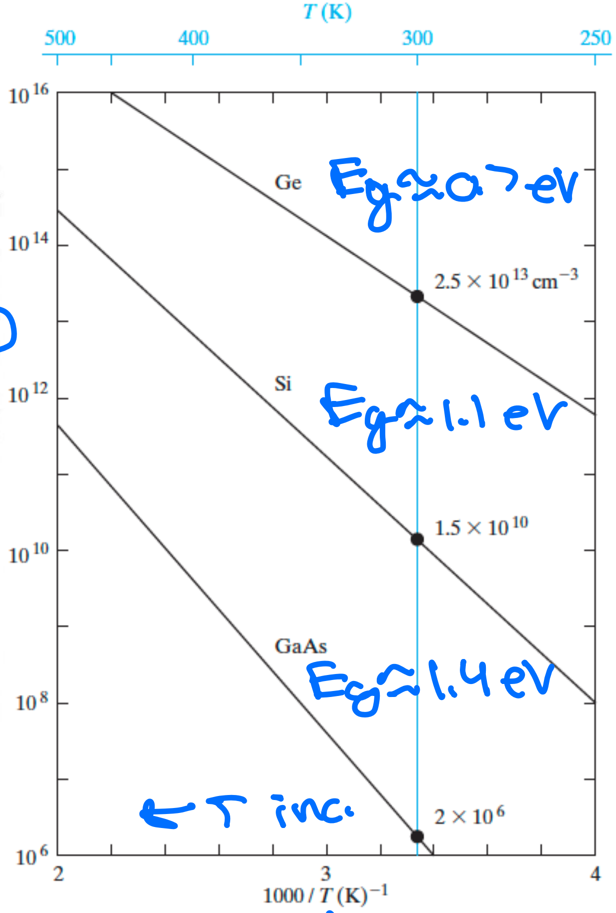
- What is this plot neglecting?

Temp. dependency of N_c, N_v, and E_g



Si	
T(°C)	n_i(cm^{-3})
0	8.86×10^8
5	1.44×10^9
10	2.30×10^9
15	3.62×10^9
20	5.62×10^9
25	8.60×10^9
30	1.30×10^{10}
35	1.93×10^{10}
40	2.85×10^{10}
45	4.15×10^{10}
50	5.97×10^{10}
300 K	1.00×10^{10}

GaAs	
T(°C)	n_i(cm^{-3})
0	1.02×10^5
5	1.89×10^5
10	3.45×10^5
15	6.15×10^5
20	1.08×10^6
25	1.85×10^6
30	3.13×10^6
35	5.20×10^6
40	8.51×10^6
45	1.37×10^7
50	2.18×10^7
300 K	2.25×10^6



inverse temp. plot

Sources: E. Pop ECE 340 Slides, Textbook

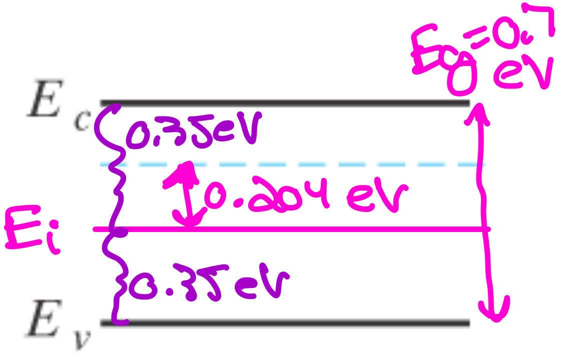
Problem: Calculating Intrinsic Carrier Concentration

kT for $T=300\,K$ ≈ 26 meV

- If we know n_i and T, we have two unknowns: (E_F-E_i) n-type and the carrier concentration. If we know one of the two, we can solve for the other:

$$n_0 = n_i e^{(E_F - E_i)/kT}$$
$$p_0 = n_i e^{(E_i - E_F)/kT}$$

n-type



(b) n-type

- Calculate and show position of the Fermi level in doped Ge (10^{16} cm^{-3} n-type) at -15 ^{o}C, using previous plot $-15^\circ C + 273 = 258\,K$

$n_i(258\,K) \approx 10^{12}\ cm^{-3}$

Assume full ionization ($T > 50$-$100\,K$)

$n_0 \approx N_d = 10^{16}\ cm^{-3}$

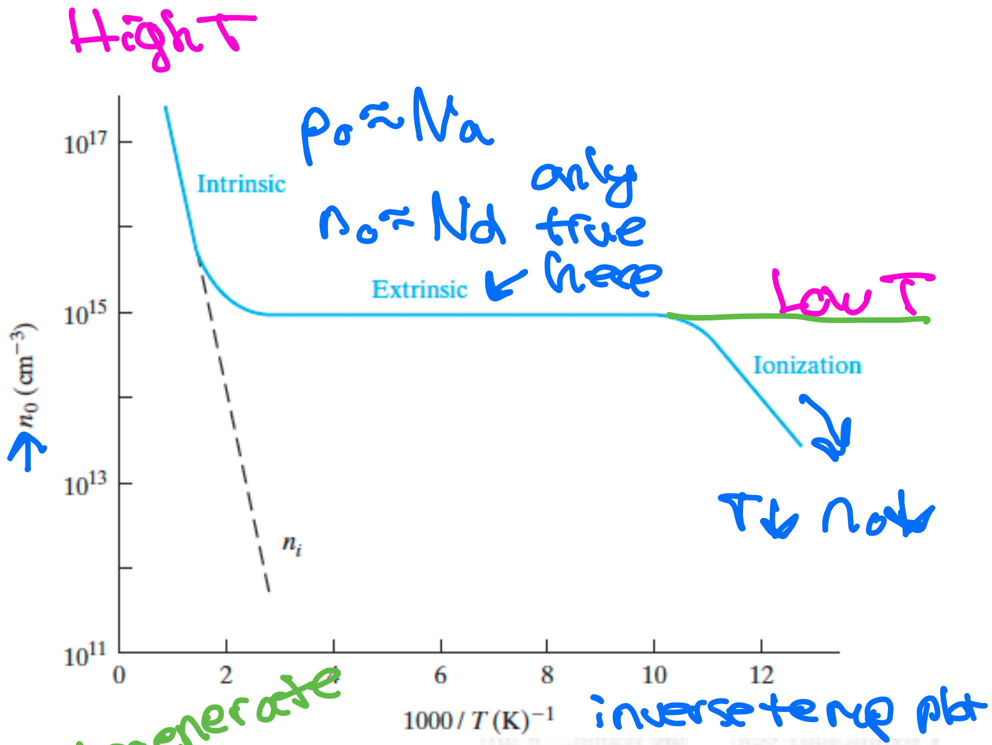
$E_F - E_i = kT \ln\left(\frac{n_0}{n_i}\right) = (8.62\times10^{-5}\ eV/K)(258\,K)\ln\left(\frac{10^{16}\ cm^{-3}}{10^{12}\ cm^{-3}}\right)$

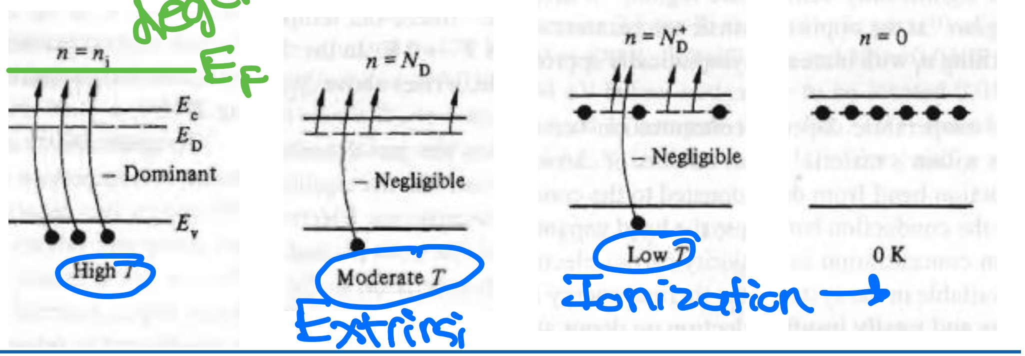
$$\boxed{E_F - E_i = 0.204\ eV}$$

Sources: E. Pop ECE 340 Slides, Textbook

Intrinsic Carrier Concentration: Temperature Regions

- Ionization **Region:** At Low T's, some donors are ionized. By ~100 K all are ionized.
- Extrinsic **Region:** At Moderate T's, $n_0 \approx N_d$ since all donors are ionized and one e$^-$ obtained for each donor atom
- Intrinsic **Region:** At High T's, # of intrinsic carriers is very high and exceeds Nd

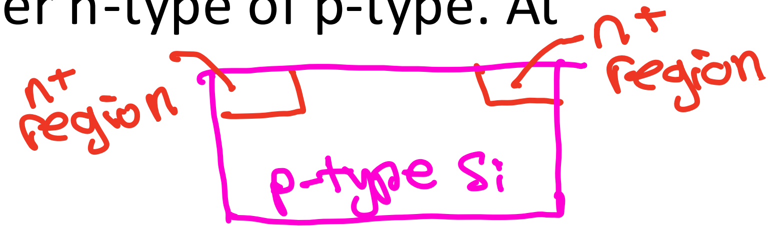




Sources: E. Pop ECE 340 Slides, Textbook

Compensation

- So far, we have assumed material is doped either n-type of p-type. At moderate temperatures: (extrinsic region)
  - $n_0 \approx N_d$
  - $p_0 \approx N_a$



- Sometimes we dope semiconductors with both donors and acceptors. This I called compensation.
- You can even start with p-type material and convert a portion of it n-type by adding enough donors
- This is a technique frequently employed to make complex devices.
- Essentially, an acceptor can effectively negate the effect of a donor!

Sources: E. Pop ECE 340 Slides, Textbook

Charge Neutrality

- What if we introduce $N_d = N_a$?
  - The material once again becomes intrinsic and $n_0 \approx p_0 \approx$ n_i.

- So far we have seen four types of charged species in a semiconductor:
  - Electrons (−)
  - Holes (+)
  - Positive donor ions $N_D^+ \rightarrow$ given up an e^-
  - Negative acceptor ions $N_A^- \rightarrow$ accepted an e^-

$$\overset{(-)}{n} + N_a = \overset{(+)}{p} + N_d$$

- In general, this is because we must have charge neutrality in the material
- Any semiconductor material is electrostatically neutral
  - Positive charge = negative charge

Sources: E. Pop ECE 340 Slides, Textbook

Carrier Concentrations in Compensated Semiconductors

- The more detailed equations for finding the carrier concentrations given values for N_a, N_d:

$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$p = \frac{N_a - N_d}{2} + \left[\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2\right]^{1/2}$$

- How do these simplify when $N_d \gg N_a$?

$$n = N_d \quad \text{and} \quad p = n_i^2/N_d$$

- When is the >> approximation valid?

At least 1 order of magnitude difference between N_d and N_a

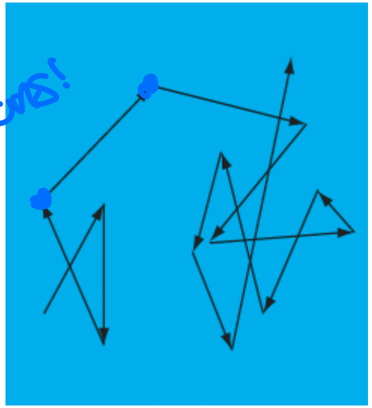
Sources: E. Pop ECE 340 Slides, C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Carrier Movement: No E-field

- So far, we've examined the effect of doping and temperature on carrier concentrations. We have assumed there is no E-field

- We know carriers are 'free' to move around. Charge carriers are in constant motion. What does this look like? Random (Brownian) motion

collisions!



- Instantaneous velocity is given by thermal energy:

$$v_{th} = \sqrt{\frac{3kT}{m^*}}$$

- What is the *net* motion of carriers? 0

- There is no preferred direction of motion for a group of carriers and no net current flow.

- Mean free time between collisions ~0.1 ps. Mean free distance a few tens of nm's.

$1 nm = 10^{-9} m$

Sources: E. Pop ECE 340 Slides, Textbook

Carrier Movement: With E-field

- Typically, we need an E-field to make use of semiconductors in devices

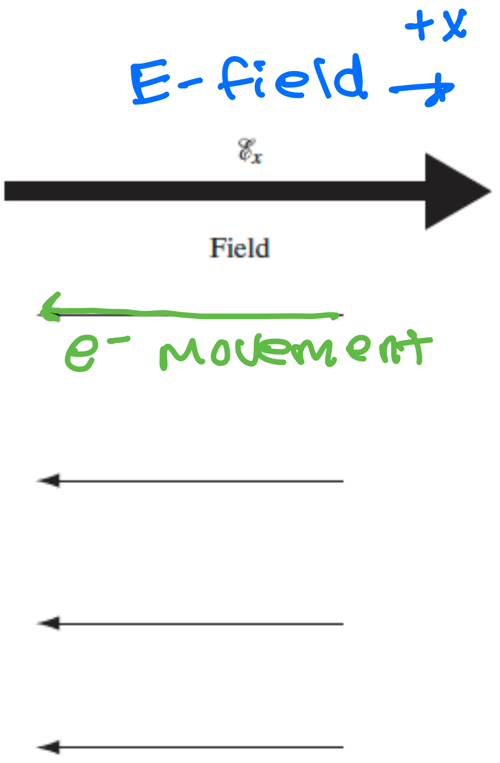
- Let's imagine we apply an E-field in the +x -direction

- What force do e^- experience?

$$F = -qE$$

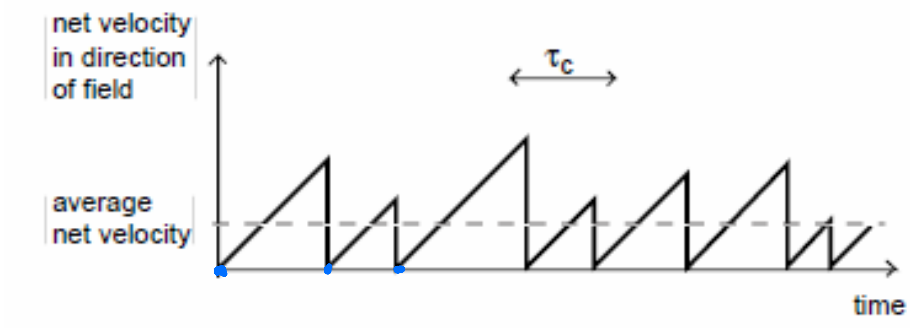
- Is there net motion of e^-?

Yes, (-)x-direction



Drift

- Drift is the motion of charge carriers caused by an E-field
- Individual e$^-$ velocity is randomized, but the average velocity is non-zero



- We can assume a mean free time between collisions, τ_c, and that each e$^-$ loses its entire drift momentum, p, after each collision:

$$p = m_n^* v_n = -qE\tau_C$$

$$v_n = -\frac{qE\tau_C}{m_n^*}$$

Sources: E. Pop ECE 340 Slides, C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Drift Velocity and Mobility

- The drift velocity is usually written with a proportionality constant,

$$\boxed{v_n = -\mu_n E}$$

- And for holes,

$$\boxed{v_p = \mu_p E}$$

- We call this proportionality constant the mobility, μ

$$\boxed{\mu_n = -\frac{q\tau_C}{m_n^*}} \qquad \boxed{\mu_p = \frac{q\tau_C}{m_p^*}}$$

- Describes the ease with which carriers drift in a semiconductor.

- Very important quantity!

- Units? $cm^2/V \cdot s$

Sources: E. Pop ECE 340 Slides, C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Mobility

- What are the roles of τ_C and m^*?
  - If m↓ "lighter" particle means μ… goes up
  - If $\boldsymbol{\tau}_{\mathrm{c}}$↑ means longer time between collisions, so μ… goes up

- Mobilities at room temperature in lightly doped semiconductors:

TABLE 2–1 • Electron and hole mobilities at room temperature of selected lightly doped semiconductors.

	Si	Ge	GaAs	InAs
μ_n (cm²/V·s)	1400	3900	8500	30, 000 * Ideal
μ_p (cm²/V·s)	470	1900	400	500

- High mobility is desired in devices --> translates to higher speed (frequencies)

Sources: E. Pop ECE 340 Slides, C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Problem

$v_{th} = \sqrt{\frac{3kT}{m^*}} = 2.2 \times 10^7$ cm/s

Check if $v_{th} > v_{drift}$

It is!

Given $\mu_p = 470\ cm^2/V \cdot s$ for Si, what is the hole drift velocity at $E = 10^3\ V/cm$? What is $\boldsymbol{\tau_c}$ and what is the average distance traveled between collisions, i.e., the **mean free path**? Assume 300 K in the dark.

$v_p = \mu_p E = (470\ cm^2/V.s)(10^3\ V/cm) = 4.7 \times 10^5$ cm/s

$\tau_c = \frac{\mu_p m_p^*}{q} = \frac{(470\ cm^2/V.s)(0.39)(9.11 \times 10^{-31}\ kg)}{1.6 \times 10^{-19}\ C} = 0.1$ ps

Mean free path: $\tau_c v_{th} = (10^{-13} s)(2.2 \times 10^7\ cm/s)$

$= 2.2 \times 10^{-6}$ cm

$= 22$ nm

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Mobility and Scattering

Duplicate slide!

see 14

- What are the roles of τ_C and m^*?
  - If m↓ "lighter" particle means μ…
  - If τ_c↑ means longer time between collisions, so μ…
- Mobilities at room temperature of lightly doped semiconductors:

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Sources: E. Pop ECE 340 Slides, C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

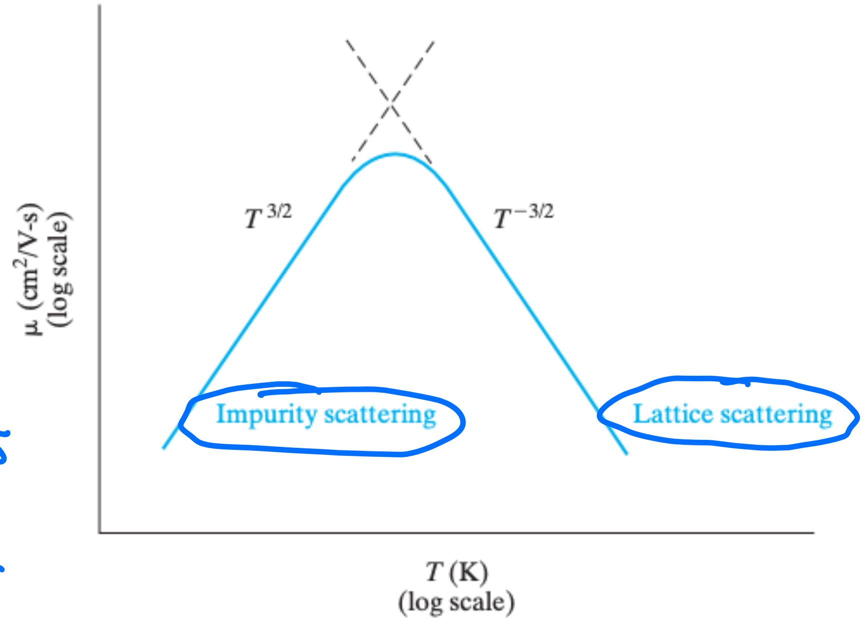
Effects of Temperature on Mobility

- What does $\boldsymbol{\mu}$ (through $\boldsymbol{\tau_c}$) depend on?
  - Phonon /lattice scattering (host lattice, like Si)
  - Ionized impurities (dopant atom) scattering

- How is scattering effected by temperature?
  - Lattice scattering increases with increased temperature, and mobility decrease $\propto T^{-1.5}$
  - Impurity scattering decrease with increased temperature, and mobility increases $\propto T^{+1.5}$

- Strongest scattering (e.g. lowest mobility) dominates total mobility:

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L} + \cdots$$



Sources: E. Pop ECE 340 Slides, Textbook

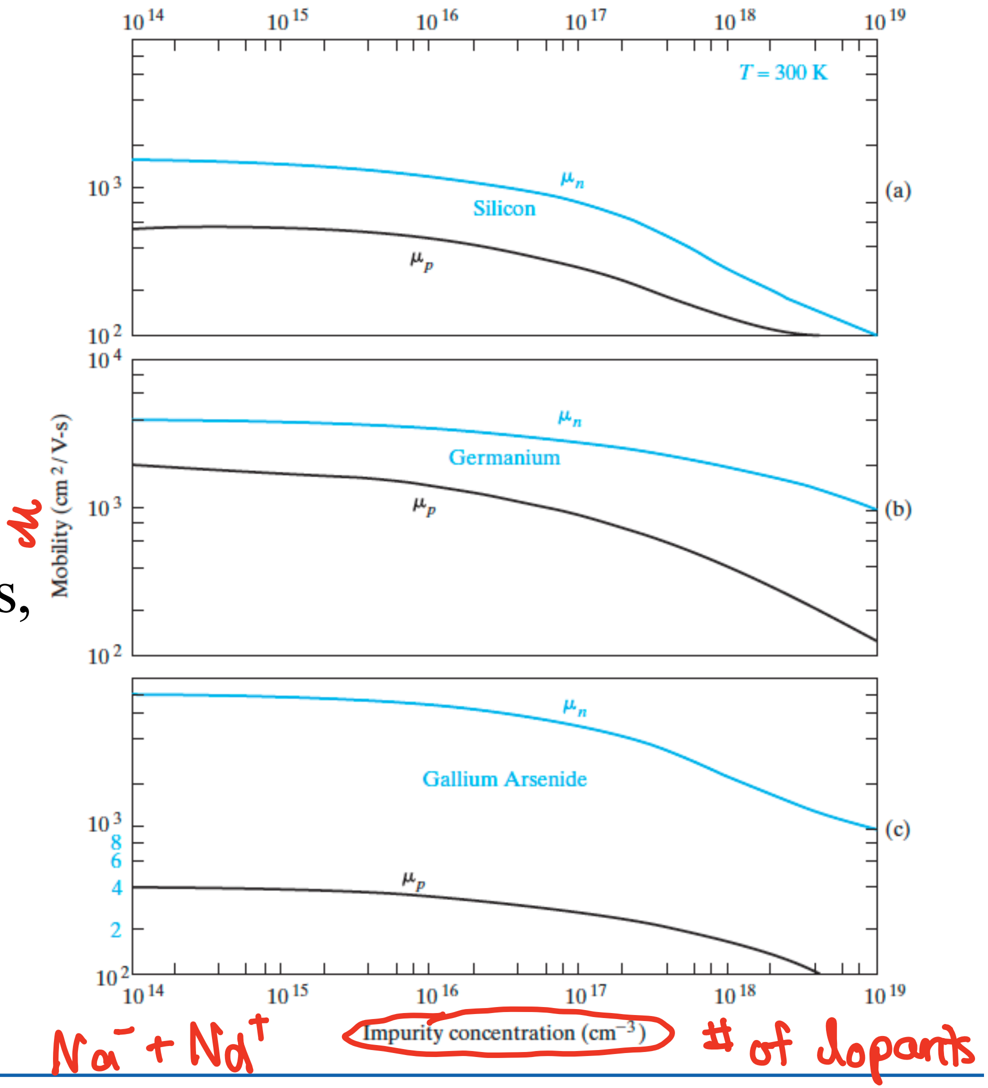
Effects of Doping on Mobility

- We expect the mobility to decrease with total impurities ($N_a^- + N_d^+$)

- Why? Increased ionized impurity scattering!

- As the concentration of dopants increases, the effects of impurity scattering are felt at higher temperatures

- In reality, mobility also depends on type of dopant



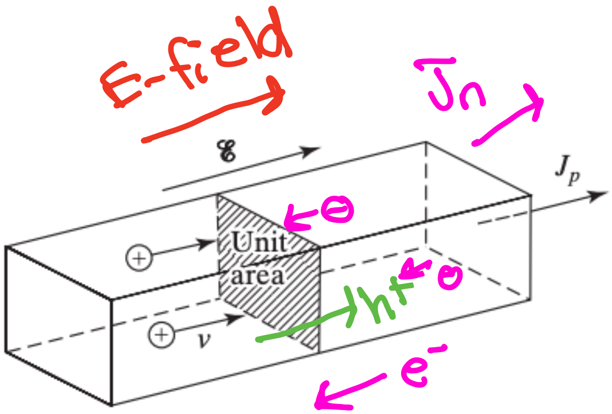
Sources: Textbook

Drift Current Components

- Recall: we have a net drift velocity of carriers.
- This is electric current! Now, we can calculate current density, J, for devices!

$$\boxed{J_n^{drift} = -qnv_{dn} = qn\mu_n E}$$

$$\boxed{J_p^{drift} = +qpv_{dp} = qp\mu_p E}$$



A p-type semiconductor bar demonstrating the concept of current density.

- So what is drift current density proportional to?
  - Carrier concentration
  - Carrier drift velocity
  - Carrier charge

- Current density, J, is the charge per second crossing a unit area plane normal to the direction of current flow (A/cm²)

* Current I: A

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Total Drift Current and Conductivity

- The total drift current density is the sum of the e$^-$ and h$^+$ components:

$$J_{\text{drift}} = J_{n,\text{drift}} + J_{p,\text{drift}} = \underbrace{(qn\mu_n + qp\mu_p)}_{\text{conductivity}}\mathcal{E}$$

Electric field

- Note: This assumes **low E-fields** (typically <10^4 V/cm in Si)

- The quantity in parenthesis is called the conductivity,

$$\boxed{\sigma = qn\mu_n + \cancel{qp\mu_p}}$$

- Recall: large ratio between majority and minority carriers
  - Usually only one term in σ

Ex. $N_d = n \approx 10^{16}$ cm^{-3} $p \approx 10^4$ cm^{-3}

Sources: Textbook

Resistivity

- Drift current density can therefore be written,

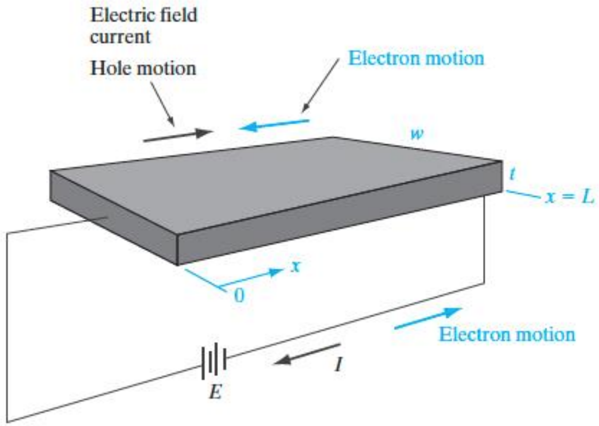
Ohm's law: current density is directly proportional to electric field!

$$\boxed{J = \sigma E = \frac{E}{\rho}}$$

- ρ is the resistivity (i.e. the inverse of conductivity),

$$\boxed{\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}}$$

Units: Ω-cm



- What about when n >> p? (n-type doped sample)
- What about when n << p? (p-type doped sample)

see previous slide! Ignore minority carrier term

- We can find the total resistance (Ω) of a bar of semiconductor with a given width, length and thickness (________):

w

L t

$$\boxed{R = \frac{\rho L}{\underbrace{wt}_{A}} = \frac{L}{wt}\frac{1}{\sigma}}$$

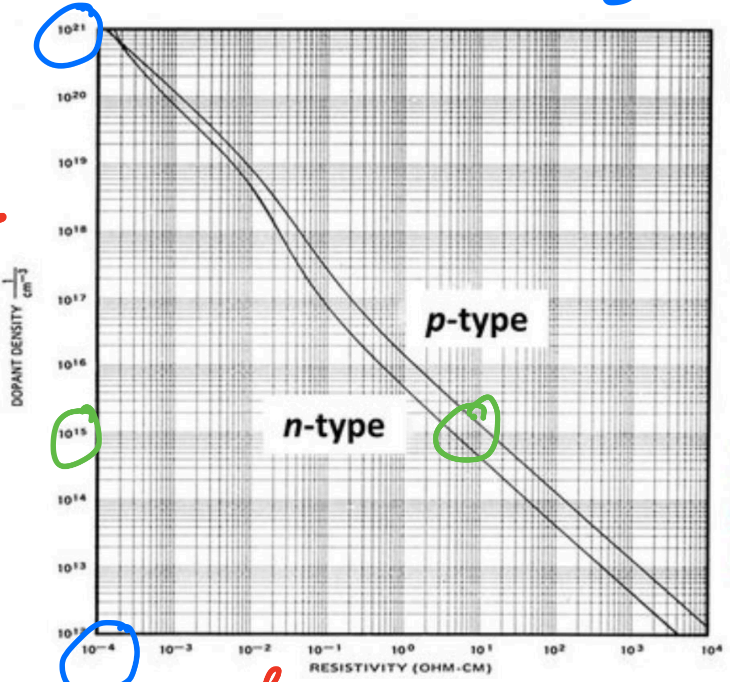
Sources: Textbook

Resistivity Dependence on Doping

- Experimentally, for Si at room temperature we get the plot below

- We have control over resistivity through doping!

Na or Nd



ρ

For n-type material:

$$\rho \cong \frac{1}{qn\mu_n}$$

For p-type material:

$$\rho \cong \frac{1}{qp\mu_p}$$

Note: This plot (for Si) does not apply to compensated material (doped with both acceptors and donors).

Sources: ECE 130/230M Slides